

## Chapter(4). Ferromagnetism and Magnetic Recording.

### (4.1) Introduction.

The magnetization density,  $\mathbf{M}$ , in most materials at room temperatures is proportional to the magnetic field,  $\mathbf{H}$ :

$$\mathbf{M} = \chi \mathbf{H}. \quad (4.1)$$

The factor of proportionality,  $\chi$ , is called the magnetic susceptibility. Since  $\mathbf{M}$ , and  $\mathbf{H}$  have the same units (Amps/meter) the magnetic susceptibility has no dimensions. Typical values of the susceptibility at room temperatures for some common substances are listed in Table(4.1). It is clear from this Table that for such non-magnetic substances the magnetization per unit volume is negligible compared with values of the impressed magnetic field,  $\mathbf{H}$ . The situation is quite different for ferromagnetic substances such as iron, nickel, or cobalt. In a ferromagnet the magnetic moments are held parallel by very strong forces called exchange forces. The magnetization per unit volume,  $M_s$ , is very large and essentially independent of applied magnetic field at temperatures low compared with a critical temperature called the Curie Temperature. The Curie temperature,  $T_C$ , is that temperature at which the magnetization in zero applied magnetic field goes to zero. The Curie temperature is material dependent. The Curie temperatures for iron, nickel and cobalt are 1044, 631, and 1393 K respectively.

**Table(4.1).** The magnetic susceptibility at room temperatures for some common non-magnetic substances. (corrected 20/06/2002)

Substance	Susceptibility in units of $10^{-6}$
Al	+20.7
Cu	-9.68
Au	-34.6
Si	-4.0
SiO <sub>2</sub>	-16.3
H <sub>2</sub> O	-9.05

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The magnetization varies slowly with temperature at low temperatures. The temperature dependence of the magnetization in iron is depicted in fig.(4.1). The temperature dependence of  $M_s$  for other ferromagnets is very similar.

If a cylindrical rod of iron were to be uniformly magnetized along its length the magnetic field strength near its end surfaces would be very large, see eqn.(3.35) and fig.(3.11). The field just outside an end face and near the cylinder axis is given by  $B_z = \mu_0 M_0 / 2$  for a cylinder whose length,  $L_d$ , is much greater than its radius,  $R$ . For iron at room temperature this field is approximately 1 Tesla. It is, however, common experience that the fields around a length of iron rod are very weak, of the order of 0.01 Teslas or less. The field outside the rod is weak because the magnetization is broken up

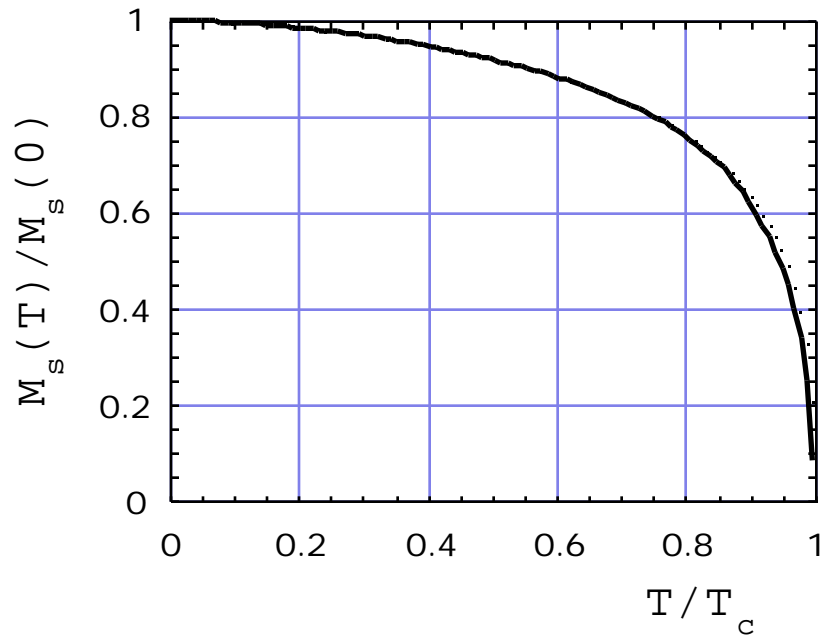


Fig.(4.1) The variation with temperature of the reduced magnetization for pure iron. The Curie temperature  $T_c = 1044$  K, and the magnetization at 0 K is  $M_s(0) = 22.1$  kOe =  $1.76 \times 10^7$  Amps/meter.

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into a very large number of small domains. Each domain carries a large magnetization,  $M_s$ , but the direction of the magnetization changes from domain to domain in such a way that the average magnetization density is very nearly zero. It can be shown that the energy due to a magnetization distribution can be calculated from

$$U_m = \frac{\mu_0}{2} \int_{\text{space}} H_m^2 dV, \quad (4.2)$$

where  $H_m$  is the magnetic field generated by the magnetic charge density  $\rho_m = -\text{div}\mathbf{M}$ . It follows that in the absence of an applied external field the domain magnetization vectors will attempt to orient their magnetization vectors so as to make  $\text{div}\mathbf{M}$  as nearly zero as possible. This tendency is called "the magnetic pole avoidance principle". The size of the magnetic domains in the absence of an applied magnetic field depends very strongly on the structure of the material (whether the specimen is a polycrystal or a single crystal), upon the concentration of impurities, and upon the presence of internal stresses. The domain dimensions in an annealed polycrystalline iron bar are of the order of 1/10 mm on a side. The domains are therefore very large compared with atomic dimensions, but are small on a macroscopic scale. In very perfect single crystalline prisms of iron in which the cubic iron axes are accurately parallel with the edges of the specimen the domains may be as long as a cm or more: see fig.(4.2).

#### (4.2) B,H Curves.

The magnetic properties of ferromagnets at a fixed temperature are often described by curves of magnetic induction  $B$  vs.  $H$ , see fig.(4.3).  $H$  is the internal magnetic field: its sources include  $\rho_m = \text{div}\mathbf{M}$  in the magnetic body as well as an externally applied magnetic field generated by a system of coils outside the magnetic

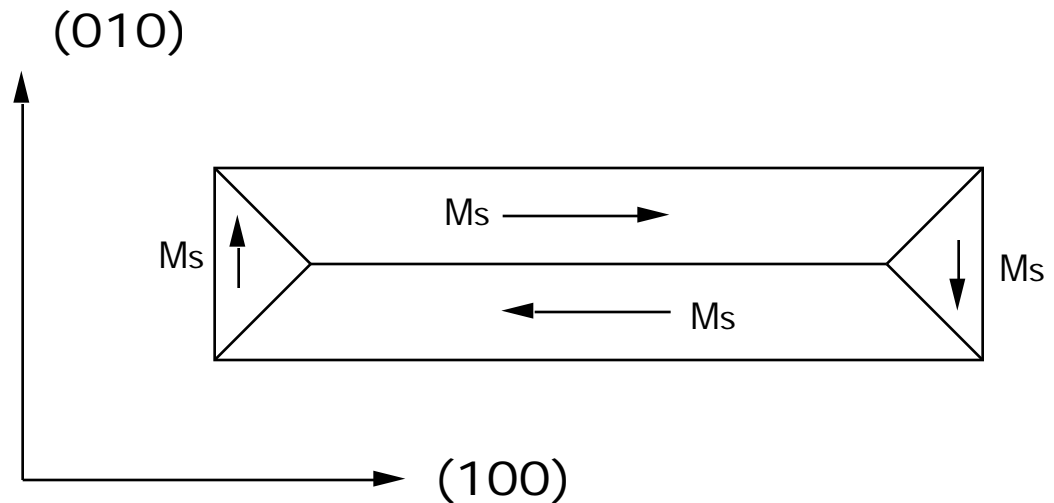


Fig.(4.2) The domain structure at room temperature in a perfect iron single crystal in which the edges of the crystal are accurately parallel with the crystalline axes. The magnetization is uniform along the z-direction (out of the paper). Simple domain structures are observed only in single crystalline specimens.

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body.  $\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M})$ , and in the simplest case the three vectors  $\mathbf{B}$ ,  $\mathbf{H}$ , and  $\mathbf{M}$  are all parallel. Starting from the fully demagnetized state ( $H=0$ ,  $M=0$ ) the magnetization increases with  $H$ , and  $B$  follows the curve labeled "virgin curve". In the virgin state with  $H=0$ , an equal number of domains have positive magnetization as have negative magnetization so that the net magnetization is zero. As  $H$  increases those domains

having a magnetization oriented along the applied field direction grow in volume at the expense of domains having a magnetization oriented opposite to the applied field direction. Eventually those domains having a component of magnetization opposed to the direction of  $H$  have been eliminated (at the point marked A in

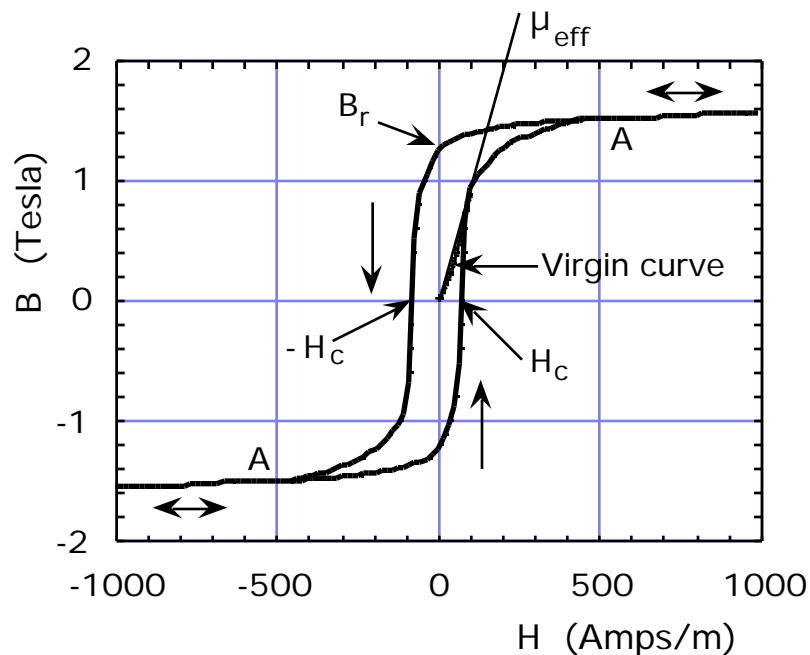


Fig.(4.3). A hysteresis loop for a polycrystalline specimen of pure iron. The details of the B-H loop are specimen sensitive. The saturation magnetization at room temperature is 2.14 Teslas. The remanent field is  $B_r = 1.22$  T, and the coercive field is  $H_c = 79$  Amps/m.

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fig.(4.3)). However, iron has a cubic crystal structure and exhibits the property that the magnetization strongly prefers to orient itself

along a direction corresponding to one of the three equivalent cubic axes. In a polycrystalline material at a field corresponding to point A in fig.(4.3) the domain magnetizations, each having a strength  $M_s$  per unit volume, are oriented at angles with respect to the applied field ranging from 0 to  $\pm 90^\circ$ . As H increases these domain magnetizations gradually rotate into the applied field direction: during this portion of the B-H loop the curve is reversible. Ultimately, the magnetization reaches the saturation value,  $M_s$ , and the magnetization density becomes uniform throughout the specimen. The field necessary to achieve the saturated state in iron,  $\sim 2 \times 10^5$  Amps/m, is very large because of the large magnetocrystalline anisotropy energy that resists the rotation of the magnetization away from a cubic axis. Very soft magnetic materials such as Supermalloy (79% Ni, 16% Fe, and 5% Mo, see Table(4.2)) have compositions corresponding to a relatively small magnetocrystalline anisotropy. The approach to saturation in such materials occurs at much lower applied fields than for iron, see fig.(4.4). It is also worth mentioning that a pure single crystal of iron for which the domain magnetizations are oriented along the cubic axes, fig.(4.2), exhibits a very large maximum effective permeability, see Table(4.2), and can be saturated in fields less than  $H = 100$  Amps/m. However, polycrystalline iron is cheap and is therefore used extensively in the construction of electromagnets and large generators.

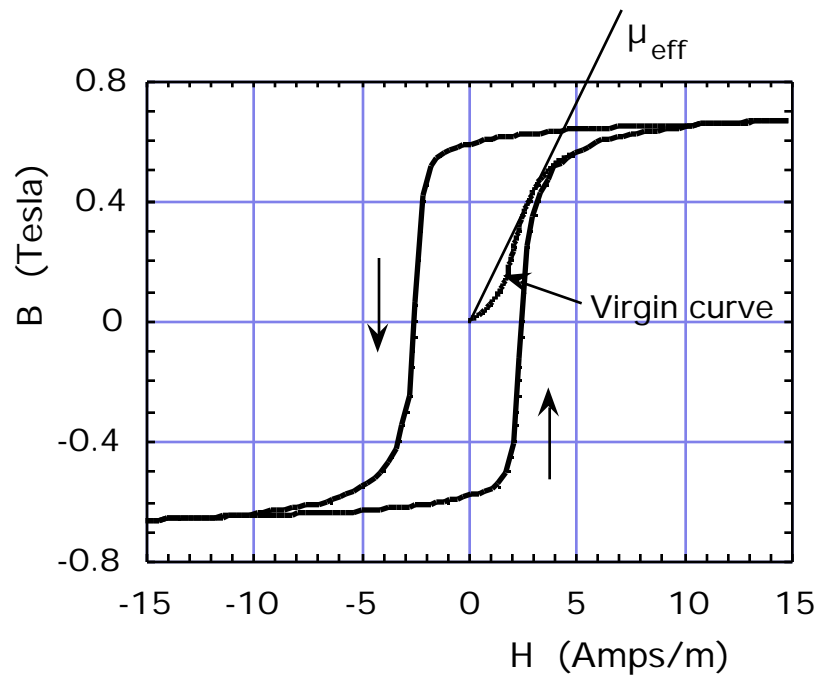


Fig.(4.4). The hysteresis loop for the soft ferromagnet 4-79 Permalloy (79% Ni,17% Fe, 4% Mo). The maximum effective permeability is 0.14 ( $\mu_R = 1.1 \times 10^5$ ), and  $H_C = 2.45$  Amps/m. The saturation field is  $B_S = 0.87$  Teslas.

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At saturation the magnetic domains have been eliminated so that the magnetization density is uniform throughout the body and has the value  $M_s$ . But  $B = \mu_0(H+M)$  so after saturation the B-field continues to increase with H, although the rate of increase of B with H becomes negligibly slow compared with the rate of increase leading up to saturation so that the variation of B is imperceptible on the scale of fig.(4.3). Upon reducing the field H after having increased it to values larger than that corresponding to point A in fig.(4.3), B

follows the upper curve in fig.(4.3) and when  $H=0$  the magnetic induction reaches the remanent field value  $B= B_r$ .  $B$  gradually falls as domains with a reversed magnetization orientation gradually reform in the body. As  $H$  is further reduced  $B$  continues to follow the upper curve and eventually  $B$  reaches zero at a negative value of  $H$  called the coercive field,  $H_C$ . Continued reduction of  $H$  ultimately leads to magnetic saturation in the negative direction with uniform magnetization having the value  $-M_s$ . As  $H$  is increased from applied field values more negative than the field corresponding to point A in the third quadrant the B-field increases along the lower curve, the magnetization becomes less negative as the number of domains having a positive magnetization increases, and ultimately  $B$  becomes positive. At the coercive field  $H_C$  the magnetic induction is zero;  $B=0$ . At sufficiently large values of the magnetic field the specimen once again becomes saturated with a uniform magnetization having the value  $M_s$ . For fields larger than 500 Amps/m, or for fields less than -500 Amps/m, the curve of  $B$  vs.  $H$  is reversible. The loop defined by the upper and lower curves in fig.(4.3) is called a major hysteresis loop. Two questions arise immediately: (1) What happens if  $H$  is decreased before point A is reached? and (2) How can the virgin state with  $H=0$ ,  $M=0$ , and  $B=0$  be attained?

If  $H$  is reduced before the reversible part of the B-H loop has been attained the B-field decreases along a minor hysteresis loop such as those shown in fig.(4.5). If a sinusoidal driving field is applied having an amplitude sufficient to drive the specimen into the

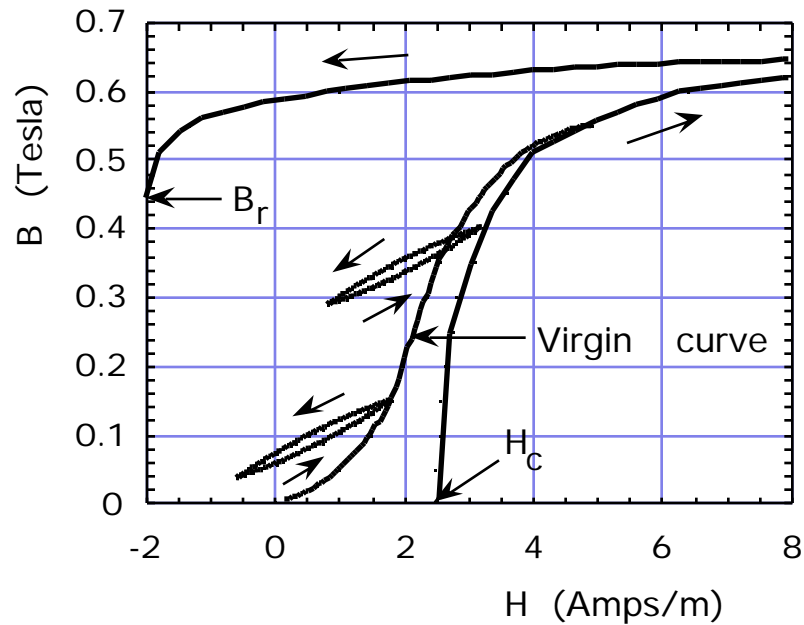


Fig.(4.5). Examples of two minor hysteresis loops in 4-79 Permalloy (see fig.(4.4). For the lowest minor loop the field was reduced after having partially traversed the virgin magnetization curve. For the upper minor loop the field was reduced after having traversed the major hysteresis curve cycle at least once.

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reversible part of the hysteresis loop the magnetic state is carried around the hysteresis loop from one extreme in the plus direction to an extreme in the negative direction many times per second. If now the amplitude of the driving field is slowly reduced to zero the hysteresis curve collapses to zero symmetrically around the origin. The specimen will be left in the virgin state in which  $B=M=H=0$ .

Important parameters associated with the hysteresis loop are (1) the remanent field,  $B_r$ , (2) the coercive field,  $H_C$ , and (3) the maximum effective permeability defined by the maximum slope of the straight line joining the origin to a point on the virgin magnetization curve as shown in figs.(4.3, 4.4). There are two major classes of ferromagnetic materials: soft ferromagnets and hard ferromagnets. Soft magnetic materials are characterized by very small values of the coercive field, see Table(4.2). For such materials the dependence of B on H is almost linear for  $H < H_C$ , and as a reasonable approximation one can write  $B = \mu_{\text{eff}} H$ . It is useful to express the effective permeability as a dimensionless number

$$\mu_{\text{eff}} = \mu_R \mu_0. \quad (4.3)$$

Pure polycrystalline iron is a soft ferromagnet characterized by  $H_C = 60$  Amps/m and  $\mu_{\text{eff}} = 0.013$ , or  $\mu_R \approx 10,000$ . There are a number of alloys that behave very nearly like perfectly soft ferromagnets, see Table(4.2).

Hard magnetic materials are characterized by large values of the coercive field and remanent field  $B_r$ , see Table(4.3). Very hard ferromagnets such as SmCo alloys, NdFeB alloys, and Strontium ferrites can be better described in terms of M vs. H. The variation of magnetization with internal magnetic field, H, is shown for a commercial Barium ferrite in fig.(4.6); only negative internal fields, H, are shown because this portion of the magnetization curve is the one required for practical applications. The hysteresis loop is very nearly rectangular, meaning that to a good approximation the

magnetization is independent of the H-field until it flips 180° at, or near, the coercive field. For very hard materials such as those listed in Table(4.3) the concept of a permeability is not very useful.

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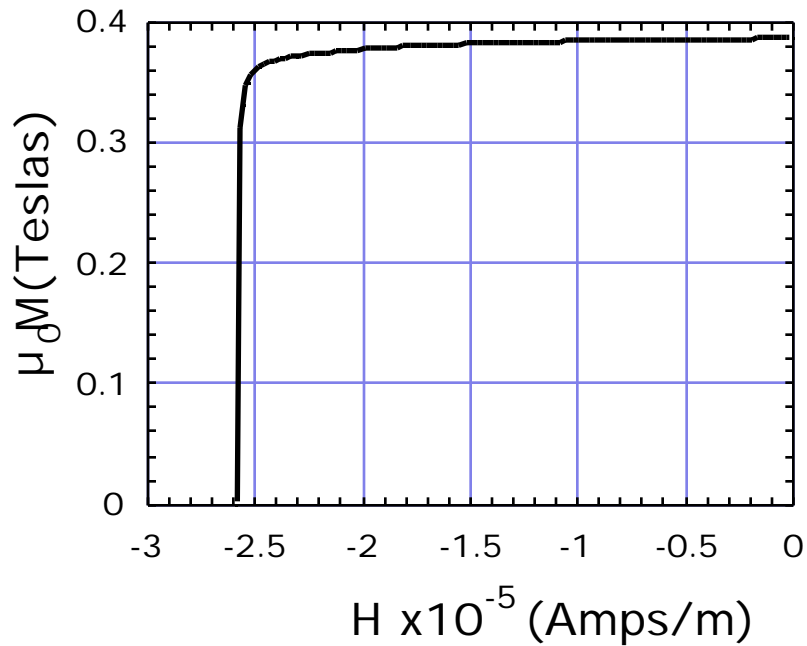


Fig.(4.6). The variation with internal magnetic field,  $H$ , of the magnetization at room temperature for a commercial Strontium Ferrite. The coercive field is  $2.59 \times 10^5$  Amps/m.

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**Warning!** Commercial hysteresis loops are usually displayed using CGS units (see Appendix(1B)). In the CGS system the fields B,M,H all have the same units, although for historical reasons the units of B,M are called Gauss whereas the units of H are called Oersteds. The conversion from CGS to MKS units is relatively simple:

$$1 \text{ Tesla} = 10,000 \text{ Gauss.}$$

$$79.6 \text{ Amps/m} = 1 \text{ Oersted.}$$

$$M \text{ in Amps/m} = [4 M(\text{in Gauss})] \times 79.6.$$

In the CGS system  $\mathbf{B} = \mathbf{H} + 4 \mathbf{M}.$

The relative permeability is the same for both systems.

**Table(4.2).** Magnetic properties of some soft magnetic materials.

Material	Saturation Field, $B_s$		Curie Temp.(C)	Coercive Field		Maximum Eff. Permeability	
	kGauss	Teslas		Oerst.	Amps/m	$\mu_R$	$\mu_{\text{eff}} = \mu_0 \mu_R$
Iron (Fig.(4.3))	21.4	2.14	770	1.0	79	6600	0.0083
Single crystal Fe	21.4	2.14	770	0.01	0.8	$1.2 \times 10^6$	1.51
Permalloy (78.5 Ni 21.5 Fe)	10.8	1.08	600	0.05	4	100,000	0.13
Supermalloy (79 Ni, 16 Fe 5 Mo)	7.5	0.75	400	0.002	0.16	$10^6$	1.26

**Table(4.3).** Magnetic properties of some commercial hard magnet materials. (See, for example, [www.dextermag.com](http://www.dextermag.com)).

Material	Residual Magnetic		Curie Temp.	Coercive Field at 300C	
	Field at 300C kGauss	Teslas		Oersted	Amps/m
Sr Ferrite	3.9	0.39	450	3250	$2.59 \times 10^5$
Samarium Cobalt	10.7	1.07	820	18000	$1.43 \times 10^6$
Sintered NdFeB	12.9	1.29	310	12900	$1.03 \times 10^6$

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#### (4.3) Measuring the B-H Loop.

It is relatively easy to measure the magnetic flux density,  $B$ , in a specimen. It is only necessary to wind a few turns of wire closely around a specimen and to measure the emf developed across the coil terminals as an external field  $B_0$  is changed with time, see fig.(4.7). The emf across the coil terminals is given by Faraday's law:

$$V(t) = NA (dB/dt), \quad (4.4)$$

where  $N$  is the number of turns on the coil and  $A$  is the cross-sectional area of the specimen in  $\text{m}^2$ . Upon integration of the voltage signal starting from a known initial condition ( $B=0$  at  $t=0$  say) one obtains  $B$  inside the specimen corresponding to a particular value of the applied field  $H_0 = B_0/\mu_0$ . In this way one can trace out the hysteresis loop of  $B$  vs  $B_0$  as the specimen is saturated first in one direction and then in the other direction. Unfortunately the hysteresis loop so obtained is not what is wanted: it depends more on the geometry of the specimen than on its intrinsic magnetic properties.

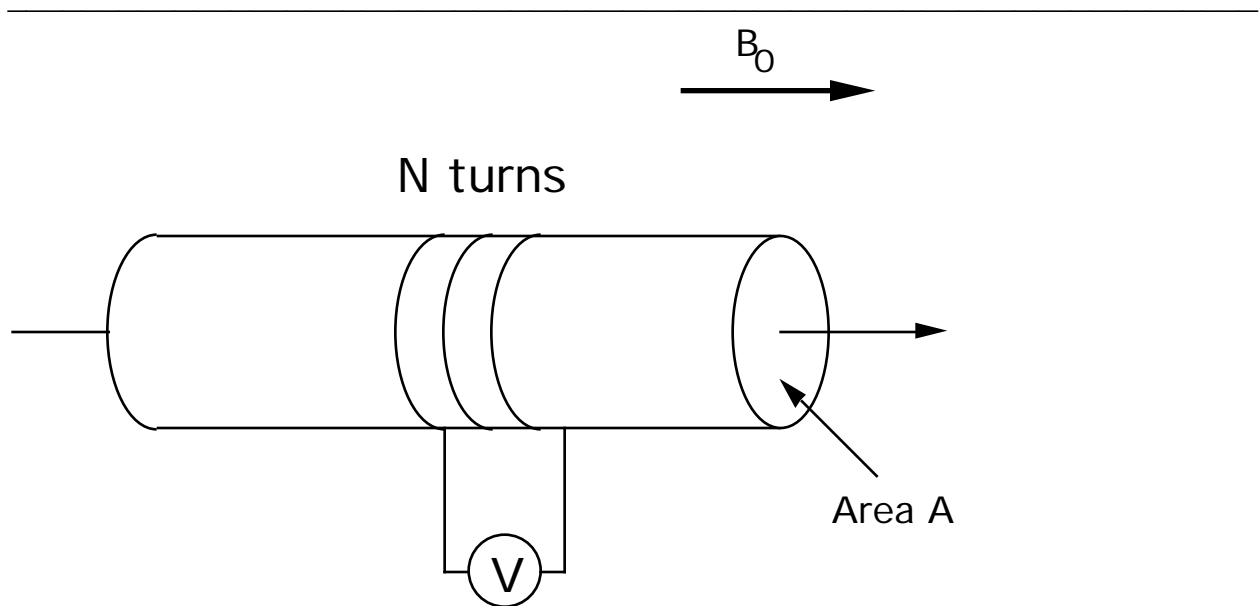


Fig.(4.7). Device for measuring the magnetization density  $B$  inside a cylinder having a cross-sectional area  $A$ . A coil of  $N$  turns is connected to a voltmeter  $V$ .

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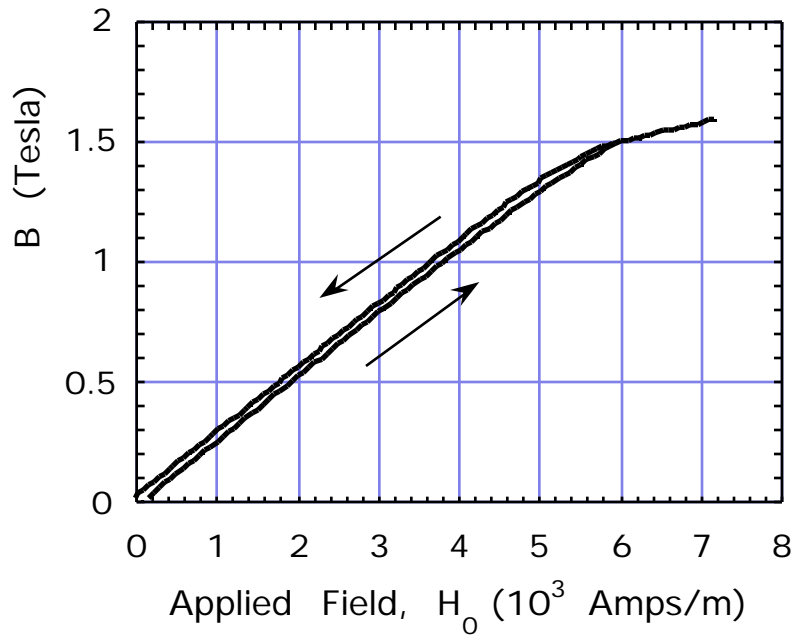


Fig.(4.8). A plot of  $B$  inside a long iron rod vs the externally applied magnetic field  $H_0 = B_0/\mu_0$ . The length to diameter ratio is 25 corresponding to a demagnetizing coefficient  $N_z = 0.00467$ .

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What is wanted is the variation of  $B$  inside the cylinder with the value of  $H$  inside the cylinder. But  $H$  inside the cylinder is the sum of the applied field  $H_0 = B_0/\mu_0$  plus the contribution generated by the magnetic pole density distribution  $\mathbf{M} = -\text{div}\mathbf{M}$ . This pole field very nearly cancels out the applied field  $H_0$  in a material having a large permeability. The net result is that the curve of  $B$  vs  $H_0$  measures an effective demagnetizing coefficient for the body under test, and does

not provide a satisfactory measure of an intrinsic magnetic property of the material of the test body. In order to see how this comes about consider a particular example: consider a cylindrical bar whose length is 25 times its diameter (25 cm long by 1 cm in diameter, for example). Let this bar be characterized by the hysteresis loop shown in fig.(4.3). The pole field inside this bar can be approximated by  $H_D = -N_z M$ , where  $N_z$  is the demagnetizing factor for an ellipsoid of revolution having the same length to diameter ratio as the cylinder;  $M$  is the magnetization density in the bar. The demagnetization factor for an ellipsoid of revolution having a length to diameter ratio of 25 is  $N_z = 0.00467$ ; see fig.(2.18). Inside the rod one has  $B/\mu_0 = H + M$ . Given the co-ordinates of a point on the B-H loop one can calculate the magnetization,  $M$ . Consider the point in fig.(4.3)  $B = 1.0$  Tesla and  $H = 110$  Amps/m. For this point  $B/\mu_0 = 1.0 / (4 \times 10^{-7}) = 0.796 \times 10^6$  Amps/m. Thus  $H$  is negligible and  $M = 0.796 \times 10^6$  Amps/m. The resulting pole field is  $H_D = -N_z M = -3.72 \times 10^3$  Amps/m. In order to obtain a net value  $H = +110$  Amps/m it is necessary to apply a field  $H_0 = 3.72 \times 10^3 + 110$  Amps/m for a total field  $H_0 = 3.83 \times 10^3$  Amps/m. In this same way one can calculate  $B$  vs  $H_0$  for all of the points on the hysteresis loop. The results of such a calculation are shown in fig.(4.8). The most obvious result is that  $B$  is nearly a linear function of the applied field  $H_0$  for  $H_0$  less than 5800 Amps/m; the slope of the line corresponds to a relative permeability  $\mu_R = 212$  (NB.  $1/N_z = 214$ ). Moreover, the hysteretic behaviour has been reduced to a very small value: approximately 0.05 Tesla in  $B$ . It is easy to show that if the material properties are such that  $B = \mu_0 \mu_R H$  then for large  $\mu_R$  one has

$B = \mu_0 H_0 / N_z$ ; ie. a straight line having a slope corresponding to  $\mu_R = 1/N_z$ . The argument runs as follows:

$$B = \mu H = \mu_0 (H + M),$$

$$\frac{\mu}{\mu_0} H = (H + M),$$

so

$$M = (\mu_R - 1)H.$$

Since the pole field is  $H_D = -N_z M$ , the applied field must overcome this pole field and supply an additional  $H$ . Therefore

$$H_0 = N_z (\mu_R - 1)H + H,$$

or

$$H = \frac{H_0}{[N_z (\mu_R - 1) + 1]},$$

and

$$B = \frac{\mu_0 \mu_R H_0}{[N_z (\mu_R - 1) + 1]}.$$

Upon dividing through by  $\mu_R$ , and taking the limit such that  $\mu_R \gg 1/N_z$ , one obtains

$$B \cong \mu_0 H_0 / N_z. \quad (4.5)$$

The point is that in order to measure the intrinsic response of a soft magnetic material it is necessary to avoid spatial variations in the magnetization that give rise to magnetic pole fields. This can be done by using a specimen having the topology of a ring, fig.(4.9). This ring can be wound with a uniformly wound primary coil of  $N_p$

turns used to generate the applied field,  $H_0$ , plus a secondary coil of  $N_s$  turns used to measure the flux density in the specimen. There

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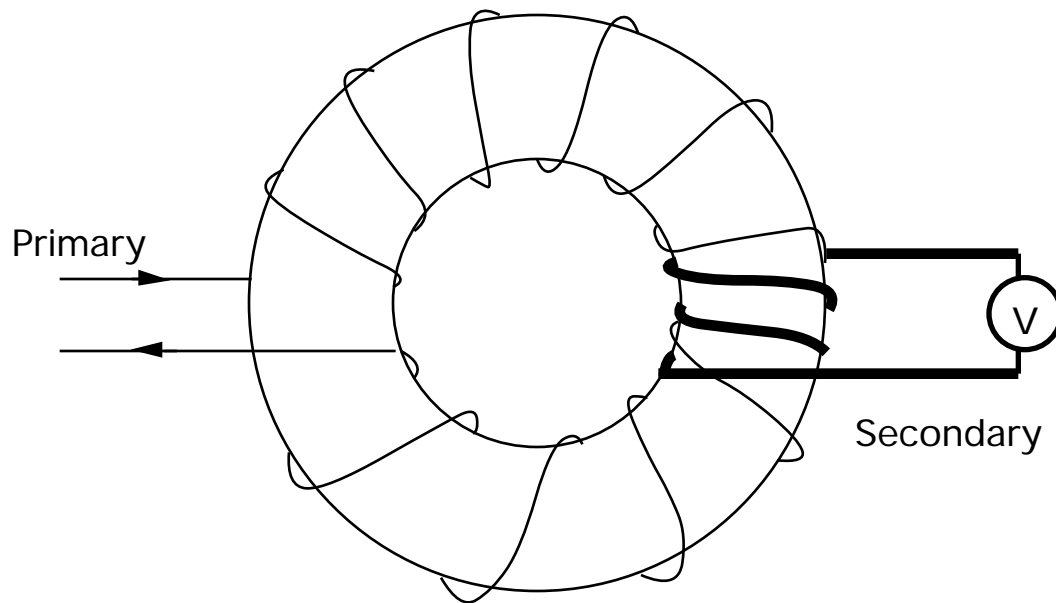


Fig.(4.9). A ring shaped specimen used to measure the intrinsic magnetic properties of a soft magnetic material. The primary winding of  $N_p$  turns is used to generate the field  $H$ . The secondary coil of  $N_s$  turns is used to measure the field  $B$  in the ring.

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are no magnetic poles if  $M$  is uniform around the ring, therefore the field in the material is just  $H = N_p I / L$ , where  $I$  is the primary coil current in Amps and  $L$  is the length in meters measured along the

centerline of the ring. The B-field can be calculated from the emf developed across the secondary windings as the primary current is changed; according to Faraday's law

$$V = N_s A (dB/dt)$$

where A is the cross-sectional area of the ring.

#### **(4.4) Digital Magnetic Recording.**

A magnetic hard disc for use as a magnetic memory storage device for a computer consists of a very smooth circular substrate upon which has been deposited a very thin coating of a magnetic cobalt alloy 50 nm or less thick. This disc is rotated at a very high rate. The remanent magnetization of this magnetic thin film is approximately  $B_R = 1/2$  Tesla, and the coercive field is approximately  $10^5$  Amps/m. The magnetization lies in the plane of the disc and contains many small, oblong regions in which the magnetization is oriented either parallel or antiparallel to the disc velocity. These magnetization regions are written into the disc magnetization by means of a write head: an extremely simplified drawing of a write head is shown in fig.(4.10). The write head is basically an electromagnet constructed of a soft magnetic permalloy yoke (the saturation field is 1 Tesla and the coercive field is  $H_c \cong 4$  Amps/m). This electromagnet is driven by the current through a single turn. The yoke contains a narrow gap,  $g$ , approximately 50 nm wide. The write head "flies" over the surface of the disc at an altitude of

approximately 25 nm, and the magnetic film on the disc is magnetized by the fringing field produced at the magnet gap. A field of approximately 3 times the coercive field is used to write magnetization regions into the disc magnetic film that are either parallel or antiparallel to the disc velocity. The spatial dependences of the fringing field components near the gap are given in the Karlqvist approximation by:

$$H_x = \frac{B_g}{\mu_0} \text{Tan}^{-1} \left[ \frac{y g}{x^2 + y^2 - (g^2/4)} \right], \quad (4.6a)$$

$$H_y = \frac{B_g}{2} \ln \left[ \frac{(x-g/2)^2 + y^2}{(x+g/2)^2 + y^2} \right], \quad (4.6b)$$

where  $B_g$  is the B-field in the middle of the gap region, and  $g$  is the gap width: the co-ordinate axes are shown in fig.(4.10b). Bits of information are stored as magnetization reversals (also called flux reversals). It is only at those places where the magnetizations are directed opposite to one another that the fringing field is large enough to be detected by the read head. This is illustrated in fig.(4.11). The absence of a flux reversal is taken to be a zero; the presence of a flux reversal is taken to be a 1. The magnetization profile for a typical run of data might look like that shown in fig.(4.12). In practice, each data byte of input is stored using a complex code that uses more than the nominal 8 bits per byte in order to build in the capability to detect and correct errors.

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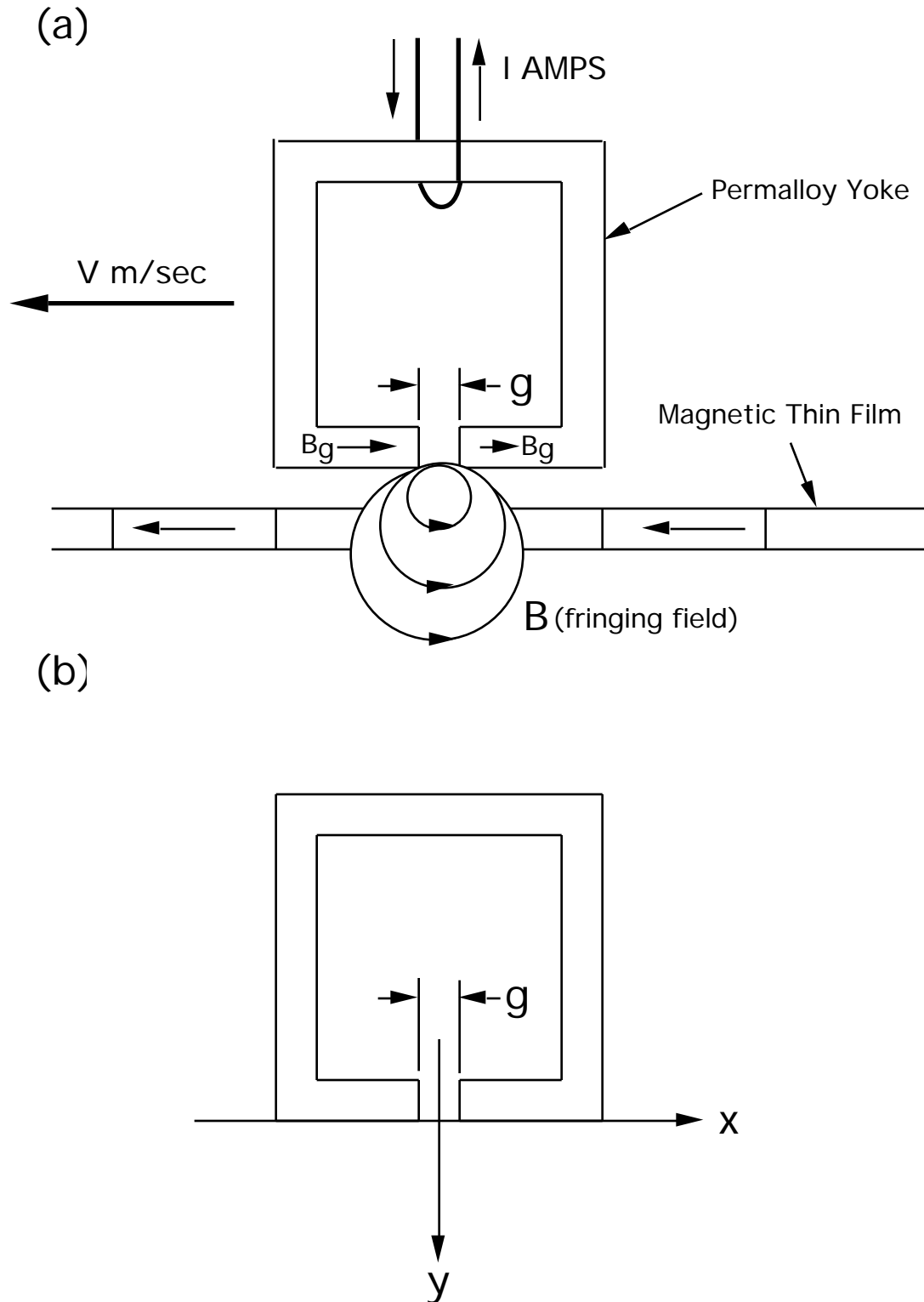


Fig.(4.10) Schematic representation of a hard disc drive write head.

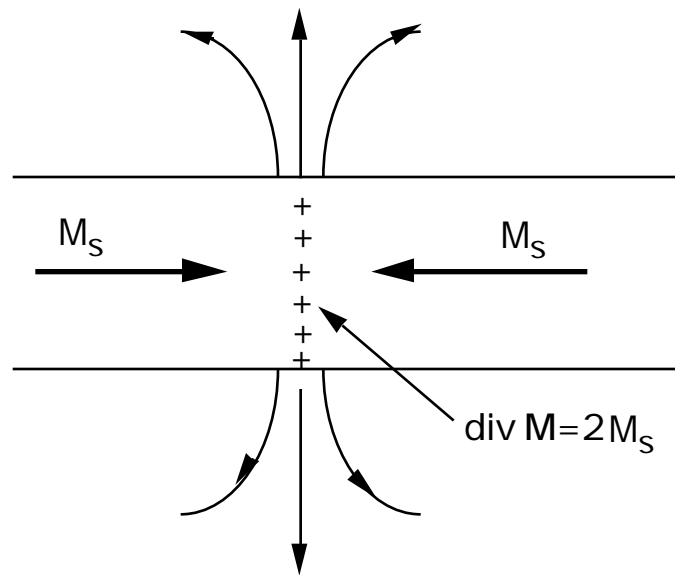


Fig.(4.11). At the juncture between two regions of oppositely directed magnetizations there is a large surface charge density, and this gives rise to large fringing fields.

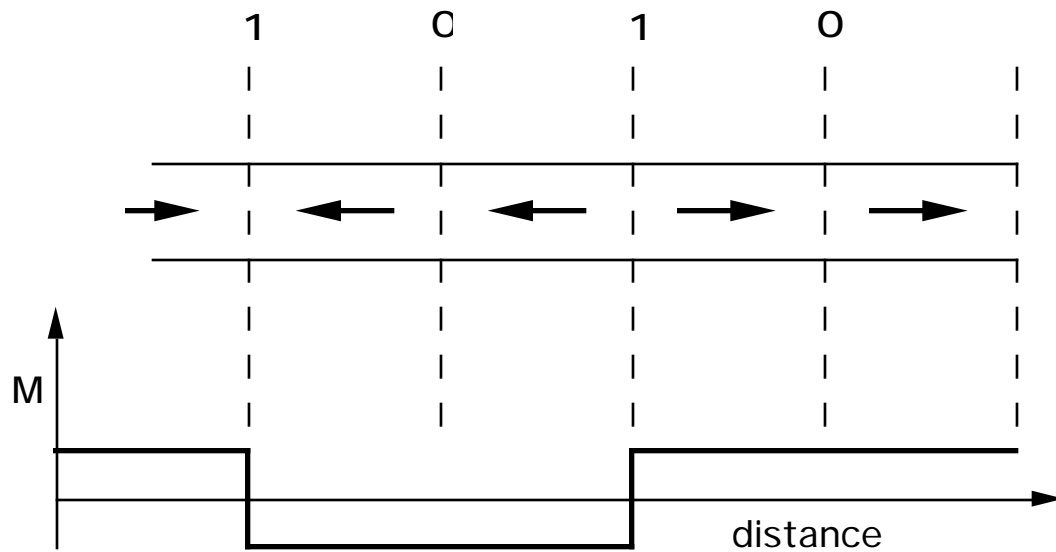


Fig.(4.12). Example of encoded data bits on a hard disc drive.

Modern read heads use a complicated structure of thin films. The magnetic field due to a magnetization change on the hard disc is detected by means of a change in resistance of a magnetoresistance element. Write and read heads are combined in a single write/read unit. A description of these units, as well as a description of the magnetoresistive element that is at the heart of the read head is contained on the IBM web site at

[www.storage.ibm.com/oem/tech/eraheads.htm](http://www.storage.ibm.com/oem/tech/eraheads.htm)

As of November 1999 IBM has demonstrated a hard disc drive having the capability to store  $3.5 \times 10^{10}$  bits per square inch using 522,000 bits per inch and 67,300 tracks per inch. This means that each magnetization cell is only 49 nm long by 377 nm wide. The disc spins at 10,000 revolutions per minute, the seek time is 4.9 msec, and information is read in and out at the rate of  $18 \times 10^6$  bytes per second. The uncorrected error rate is  $1:10^8$ ; after correction this error rate decreases to less than  $1:10^{12}$ .

#### **Further Reading.**

S. Chikazumi, "Physics of Magnetism". John Wiley and Sons, New York, 1964.

John C. Mallinson, "The Foundations of Magnetic Recording", Second Edition. Academic Press, San Diego, 1993.

John C. Mallinson, "Magneto-Resistive Heads". Academic Press, San Diego, 1996.

## Appendix(4A).

The field in the gap of an electromagnet.

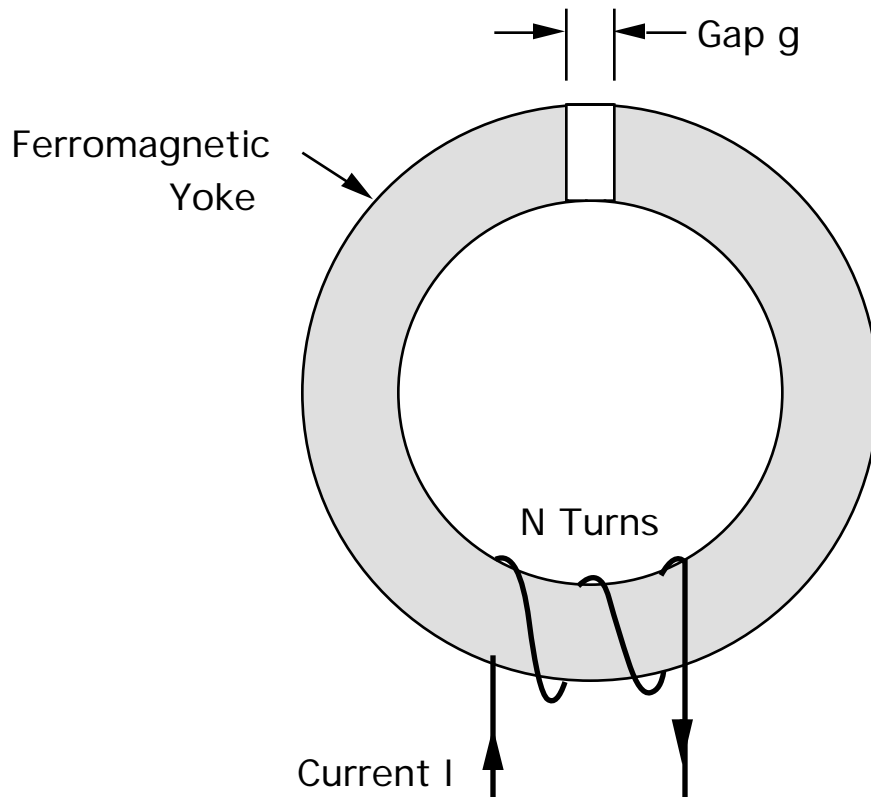


Fig.(4A.1). An electromagnet consisting of a soft ferromagnetic yoke wound with  $N$  turns of wire and containing a gap of width  $g$  meters. The length of the ferromagnetic yoke along its center line is  $L$  meters.

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Consider the electromagnet shown in fig.(A1). As a first approximation let the B-field in the ferromagnetic yoke be uniform with the same value B Teslas everywhere. The field in the gap measured along the centerline will also be B to a good approximation. This follows from the Maxwell equation  $\text{div}\mathbf{B}=0$  which requires the normal component of  $\mathbf{B}$  to be continuous across a material discontinuity. If the gap field is B then the H-field in the gap is  $H_g = B/\mu_0$ . The permeability of free space,  $\mu_0 = 4 \times 10^{-7}$ , is a small number therefore  $H_g$  will be quite large: if  $B=1.0$  Tesla then  $H_g = 7.96 \times 10^5$  Amps/m. This H-field is much larger than H within the soft ferromagnetic yoke material. For example, in iron the field H cannot exceed 100 Amps/m if  $B= 1.0$  Tesla, see fig.(4.3). According to another Maxwell equation for the static magnetic field

$$\text{curl}\mathbf{H} = \mathbf{J},$$

or

$$\oint_C \mathbf{H} \cdot d\mathbf{L} = \int_{\text{Area}} \mathbf{J} \cdot d\mathbf{A} , \quad (4A.1)$$

from Stokes' theorem where the Area of the surface integration is bounded by the curve C. Apply eqn.(4A.1) to the closed line running along the centerline of the magnet. The line integral becomes

$$L H + g H_g = N I,$$

where L is the length of the path in the ferromagnetic yoke. Given a value for the B-field one can look up the corresponding value of the

H-field from the ferromagnetic hysteresis loop. Then the current required to produce that B-field is

$$I = \frac{1}{N} [ LH + gB/\mu_0 ]. \quad (4A.2)$$

In this way one can construct a graph of B vs. I corresponding to various points on the B-H loop. It should be noted that this simple construction fails when the ferromagnet approaches magnetic saturation.