Neutron stars

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Abstract

Neutron stars exhibit conditions far from those encountered on earth, which makes them interesting objects by themselves. Moreover, the physics of compact objects like neutron stars offers an intriguing interplay between nuclear processes and astrophysical observables.

V neutronových hvězdách panují podmínky velmi odlišné od pozemských podmínek, díky čemuž jsou neutronové hvězdy zajímavými objekty samy o sobě. Mimoto, fyzika kompaktních objektů jako jsou neutronové hvězdy nabízí fascinující souhru mezi nukleárními procesy a astronomickými pozorováními.

1 Introduction

In 1054, ancient Chinese astronomers observed supernova explosion that gradually evolved into the object known as Crab Nebula nowadays. Two years after Chadwick's discovery of the neutron in 1932, Baade and Zwicky [Baade and Zwicky, 1934a, Baade and Zwicky, 1934b] proposed the idea of neutron stars, i.e., 'stars' of very high density and small radius. The first calculations of neutron star models were performed by Oppenheimer and his collaborators [Oppenheimer and Volkoff, 1939], who assumed neutron star matter to be composed of an ideal gas of free neutrons at high density.

The discovery of cosmic non-solar X-ray sources by Giacconi et al. [Giacconi et al., 1962] in 1962, however, generated new interest in the concept of neutron stars [Østgaard, 2001].

Theoretical work discussed equations of state and neutron star models, and also equilibrium properties of compact stars and star collapse. But the possibility of neutron stars (or black holes) was still not taken seriously until pulsars were discovered by a group of Cambridge astronomers in 1967. In 1967 Hewish et al. [Hewish et al., 1968] detected an astronomical object emitting periodic pulses of radio waves. This had a great impact on astrophysical research on compact stars, since the existence of stable equilibrium stars more dense than white dwarfs had already been predicted, and it had been suggested that such objects could be produced in supernova explosions.

Shortly after Gold [Gold, 1968] proposed that pulsars are in fact rotating neutron stars with surface magnetic fields of the order of 10^{12} Gauss (see Fig. 1). Such objects could account for the observed remarkable stability of the pulse period. The implied energy loss was roughly the same as the energy required to power the Crab Nebula, which led to the general acceptance of the neutron star model. The discovery in 1968 of the Crab and Vela pulsars located in supernova remnants provided evidence for the formation of neutron stars in supernova explosions.

Further work was stimulated by the discovery of pulsating compact X-ray sources by the UHURU satellite in 1971. These X-ray pulsars are neutron stars in close binary systems, accreting gas from their normal companion

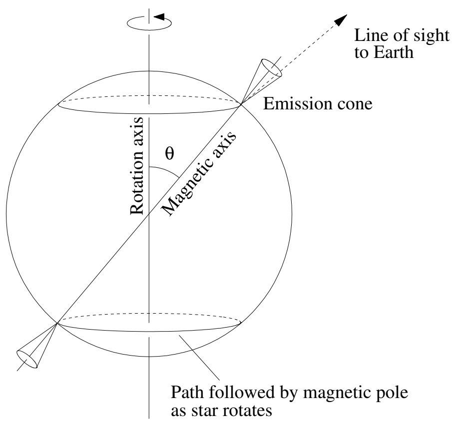

Figure 1: Model of pulsar as a rotating neutron stars. Taken from Ref. [Carroll and Ostlie, 1996].

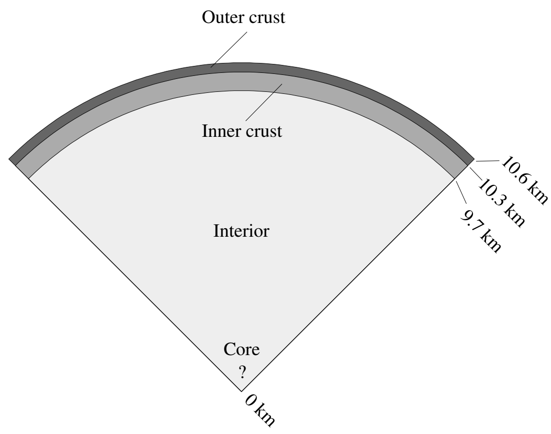

Figure 2: Model of a neutron star internal structure. Taken from Ref. [Carroll and Ostlie, 1996].

stars. The first binary pulsar was discovered by Hulse and Taylor [Hulse and Taylor, 1975], and makes it possible to measure the mass of a neutron star and, for instance, to test the existence of gravitational radiation indirectly.

The determination of an equation of state for dense matter then is central to calculations of neutron star properties, and it determines the mass range as well as the mass-radius relationship for these stars. It is also an important ingredient in the determination of the composition of dense matter and the thickness of the crust in a neutron star. The latter influences neutrino generating processes and the cooling of neutron stars [Pethick, 1992].

2 Internal structure

One of the important aspects in the study of neutron stars is the equation of state, i.e., the functional dependence of the pressure P on the density ρ. In the interior of neutron stars the nuclear matter is found at densities above the neutron 'drip' $\rho_\mathrm{d} \approx 4 \times 10^{11}\,\mathrm{g/cm^3}$, and the properties of cold dense matter and the associated equation of state are reasonably well understood at densities up to $\rho_\mathrm{n} \approx 3 \times 10^{14}\,\mathrm{g/cm^3}$. In the high-density range above ρ_n the physical properties of matter are still uncertain.

Typical neutron stars then have masses in the range $M = 1$–$2\,M_\odot$ and radii of the order of $R \sim 10\,\mathrm{km}$. The different regions of the star are described as follows (see Fig. 2):

- The *surface* for $\rho < 10^6\,\mathrm{g/cm^3}$ is a region in which the temperatures and magnetic fields expected for most neutron stars can affect significantly the equation of state.
- The *outer crust* for $10^6\,\mathrm{g/cm^3} \leq \rho \leq 4 \times 10^{11}\,\mathrm{g/cm^3}$ is a solid region in which a Coulomb lattice of heavy nuclei coexists in β-equilibrium with a relativistic degenerate electron gas.
- The *inner crust* for $4\times10^{11}\,\mathrm{g/cm^3} < \rho < 2\times10^{14}\,\mathrm{g/cm^3}$ consists of a lattice of neutron-rich nuclei together with a superfluid neutron gas and an electron gas.
- The *neutron liquid* for $2 \times 10^{14}\,\mathrm{g/cm^3} < \rho < 8 \times 10^{14}\,\mathrm{g/cm^3}$ contains mainly superfluid neutrons with a smaller concentration of superfluid protons and 'normal' electrons.
- The *core* region for $\rho > 8 \times 10^{14}\,\mathrm{g/cm^3}$ may or may not exist in some neutron stars, and will depend on whether or not kaon condensation or pion condensation occurs, or whether there is a transition to a

neutron solid or quark matter or some other phase of hyperons physically distinct from a neutron liquid in the core.

The core could, for instance, contain geometrically mixed phases of nuclear and quark matter, where the geometries may be idealized as drops, rods and slabs. There could be an inner sphere of pure quark matter surrounded by a crystalline region of mixed hadronic and quark matter. The mixed phase region then consists of various geometrical objects of the rare phase immersed in the dominant one, from hadronic drops immersed in quark matter to quark drops immersed in hadronic matter. The particle composition of these regions then should be quarks, nucleons, hyperons and leptons. A liquid of neutron star matter containing nucleons and leptons would surround the mixed phase, and a crust of heavy ions should form the stellar surface.

3 Neutron star from the microphysics point of view

As stated above, the determination of an equation of state for dense matter then is the principal task to obtain the properties of neutron star, and it determines the mass range as well as the mass-radius relationship for these stars. Let us look at a fairly simple and unrealistic yet instructive approach first applied by Oppenheimer et al. [Oppenheimer and Volkoff, 1939].

3.1 Ideal Fermi gas of noninteracting neutrons

The simplest cold, degenerate equation of state is that due to a single species of ideal (noninteracting) fermions. This corresponds to picturing the neutron star as a sphere of pure neutron matter. We assume free neutrons only, and no protons, electrons or other particles in the system.

From kinetic theory we know the pressure and energy density:

$$P = \frac{1}{3h^3}\int g f(E) p v \, \mathrm{d}^3 p, \qquad v = \frac{pc^2}{E}, \qquad g = 2, \tag{1}$$

$$\epsilon = \rho c^2 = \frac{1}{h^3}\int g f(E) E \, \mathrm{d}^3 p, \qquad E = \sqrt{p^2c^2 + m_\mathrm{n}^2 c^4}, \tag{2}$$

where m_n is the neutron mass. For an ideal fermion gas, the distribution $f(E)$ has the well-known form

$$f(E) = \frac{1}{\exp\left[(E-\mu)/kT\right] + 1}, \tag{3}$$

where $\mu = \mathrm{d}\epsilon/\mathrm{d}n$ is the chemical potential. For completely degenerate fermions ($T \to 0$, $\mu/kT \to \infty$), μ is called the Fermi energy E_F, and

$$f(E) = \begin{cases} 1, & E \le E_\mathrm{F} \\ 0, & E > E_\mathrm{F} \end{cases}. \tag{4}$$

Introducing the Fermi momentum p_F by $E_\mathrm{F} = \sqrt{p_\mathrm{F}^2 c^2 + m_\mathrm{n}^2 c^4}$, the 'relativity parameter' x and the Compton wavelength λ_n of neutron

$$x \equiv \frac{p_\mathrm{F}}{m_\mathrm{n} c}, \qquad \lambda_\mathrm{n} \equiv \frac{\hbar}{m_\mathrm{n} c}, \tag{5}$$

we can express the number density of neutrons, the pressure, and energy density as

$$n_\mathrm{n} = \frac{2}{h^3}\int_0^{p_\mathrm{F}} 4\pi p^2 \, \mathrm{d}p = \frac{8\pi}{3h^3} p_\mathrm{F}^3 = \frac{1}{3\pi^2 \lambda_\mathrm{n}^3} x^3, \qquad P = \frac{m_\mathrm{n} c^2}{\lambda_\mathrm{n}^3}\,\phi(x), \qquad \epsilon = \rho c^2 = \frac{m_\mathrm{n} c^2}{\lambda_\mathrm{n}^3}\,\chi(x), \tag{6}$$

respectively. The functions ϕ and χ are given by

$$\phi(x) = \frac{1}{8\pi^2}\left\{x(1+x^2)^{1/2}(2x^2/3 - 1) + \ln\left[x + (1+x^2)^{1/2}\right]\right\}, \tag{7}$$

$$\chi(x) = \frac{1}{8\pi^2}\left\{x(1+x^2)^{1/2}(1+2x^2) - \ln\left[x + (1+x^2)^{1/2}\right]\right\}. \tag{8}$$

Two limiting cases for this equation of state are of special interest. For rest mass density $\rho_0 \ll 6\times10^{15}\,\mathrm{g/cm^3}$ the neutrons are nonrealtivistic, while for $\rho_0 \gg 6\times10^{15}\,\mathrm{g/cm^3}$ the neutrons become extremely relativistic. The equation of state then can be written in the polytropic form $P = K\rho^\Gamma$ with K and Γ constant. For nonrelativistic case $\Gamma = 5/3$, for extremely relativistic case $\Gamma = 4/3$.

3.2 More realistic equations of state

Calculations can be done for different equations of state coupled together in different ways, i.e., calculations for previously known equations of state, or for some new equations of state. If tabulated values for pressure (or energy) are calculated or given, a new equation of state is obtained in the following way: The pressure (or the energy) is fitted by a polynomial consisting of N terms, i.e.,

$$P(n) = \sum_{i=1}^{N} c_i n^{l_i}, \tag{9}$$

where the particle density n is given in [fm^{-3}], and data for N different values of P and n are used in order to calculate the coefficients c_i. The corresponding equations are solved by matrix inversion, and we obtain a polynomial $P(n)$.

Different equations of state or polynomials are, for instance, labeled as

- MBJ, AM-5, AM-9, CHH [Mølnvik and Østgaard, 1985],
- AØØ-5 [Øvergård and Østgaard, 1991c],
- BBP-1, BBP-2, PS [Øvergård and Østgaard, 1991b],
- MIT, QCD [Øvergård and Østgaard, 1991a],
- Sierk–Nix, Quadratic [Rosenhauer et al., 1992],
- WFF-1, WFF-2, WFF-3, and FP [Bao et al., 1994].

Some resulting equations of state are shown in Fig. 3. The WFF-2 equation of state is very similar to the FP equation of state.

The equations of state valid for various ranges can be then combined as depicted in the example in Fig. 4.

4 Neutron star from the macrophysics point of view

Newton theory of gravitation fails to describe the equilibrium configuration of neutron star because the surface dimensionless potential

$$\frac{G}{c^2}\frac{M}{R} \sim 0.1, \qquad \frac{G}{c^2} \approx 0.74\times10^{-28}\,\mathrm{cm/g} \approx 1.5\,\mathrm{km}/M_\odot \tag{10}$$

shows that the general relativistic corrections are of order 10 %. The metric of spherical symmetric static spacetime in Schwarzschild coordinates reads

$$\mathrm{d}s^2 = -\mathrm{e}^{2\Phi(r)}c^2\,\mathrm{d}t^2 + \mathrm{e}^{2\Psi(r)}\,\mathrm{d}r^2 + r^2(\mathrm{d}\theta^2 + \sin^2\theta\,\mathrm{d}\phi^2), \qquad \mathrm{e}^{2\Psi(r)} = \left[1 - \frac{2m(r)}{r}\right]^{-1}, \tag{11}$$

where

$$m(r) = \int_0^r 4\pi r'^2 \rho(r')\,\mathrm{d}r' \tag{12}$$

is the 'mass inside the sphere of radius r' [Misner et al., 1973], ρ denotes the mass density. The equations of hydrostatic equilibrium and Eq. (12) take the form

$$\frac{\mathrm{d}P}{\mathrm{d}r} = -(P+\epsilon)\frac{Gmc^2 - \frac{1}{3}\Lambda r^3c^4 + 4\pi GPr^3}{rc^2(rc^2 - 2Gm - \frac{1}{3}\Lambda r^3c^2)}, \tag{13}$$

$$\frac{\mathrm{d}m}{\mathrm{d}r} = 4\pi r^2\frac{\epsilon}{c^2}, \tag{14}$$

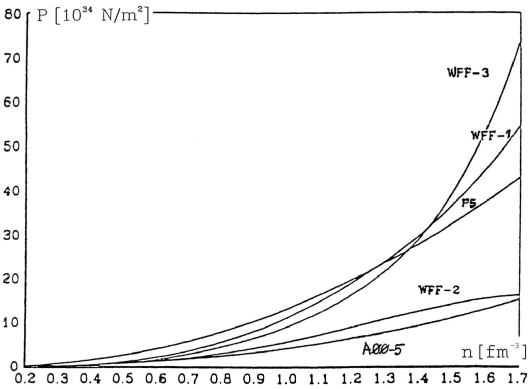

Figure 3: Pressure P as a function of density for AØØ-5, WFF-1, WFF-2, WFF-3 and PS equations of state. Taken from Ref. [Østgaard, 2001].

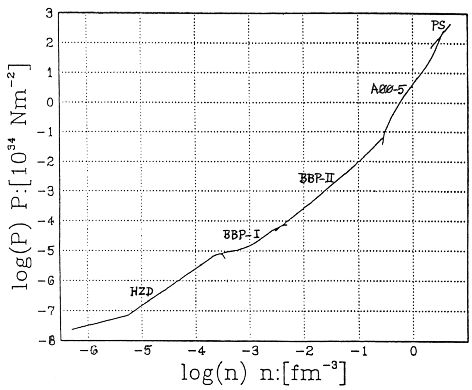

Figure 4: Pressure P as function of density for coupled HZD, BBP-1, BBP-2, AØØ-5 and PS equations of state. Taken from Ref. [Østgaard, 2001].

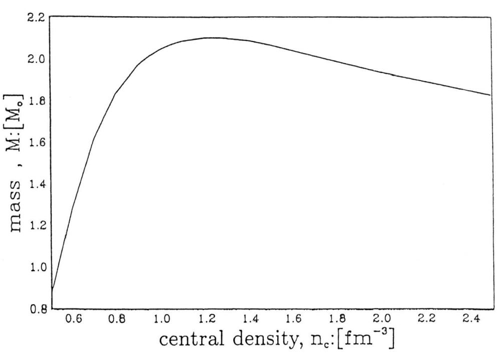

Figure 5: Mass M as function of central density ρ_c for coupled HZD, BBP-1, BBP-2, WFF-3 and PS equations of state. Taken from Ref. [Østgaard, 2001].

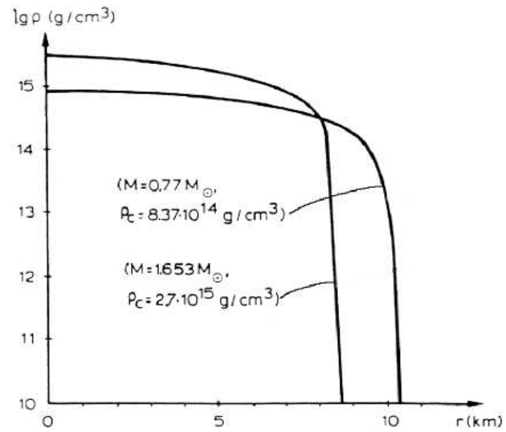

Figure 6: Pressure P as a function of the distance r from the centre of the star, calculated with the AM-9 equation of state for two different central densities ρ_c. Taken from Ref. [Østgaard, 2001].

where P denotes the pressure, $\epsilon = \rho c^2$ denotes the energy density. Eq. (13) represents the standard Tolman–Oppenheimer–Volkoff equations of hydrostatic equilibrium generalized with the cosmological constant term Λ. (In case of neutron stars, the influence of this term is negligible, however, in case of general relativistic adiabatic fluid spheres their structure can be influenced strongly by a repulsive cosmological constant for high values of the adiabatic index. For detailed account, see Ref. [Stuchlík, 2002].)

The time metric coefficient Φ is determined by

$$\frac{\mathrm{d}\Phi}{\mathrm{d}r} = \frac{m(r) + 4\pi r^3 p/c^2}{r^2\,[1 - 2Gm(r)/c^2 r]}. \tag{15}$$

Besides the above Eqs (13)–(15) we have to include the equation of state $p = p(\rho)$ described in previous section. The resulting system of ordinary differential equations is to be integrated with initial conditions

$$\rho|_{r=0} = \rho_{\mathrm{c}}, \qquad p|_{r=0} = p(\rho_{\mathrm{c}}), \qquad m|_{r=0} = 0, \qquad \Phi|_{r=0} = \text{arbitrary value}. \tag{16}$$

The surface $r = R$ is reached when the pressure (but not the pressure gradient!) vanishes. The time metric coefficient (15) then can be adjusted by adding suitable value to satisfy the condition of asymptotical flatness $\lim_{r\to 0} \Phi(r) = 0$.

5 The synthesis

Solving the system of ODEs described in previous section, we can obtain total masses and radii for different (combinations of) equations of state. Some examples are given in Figs 5–6.

In general we obtain a maximum mass of $1.6\,M_\odot < M_{\max} < 2.4\,M_\odot$, with corresponding radii $8\,\mathrm{km} < R < 13\,\mathrm{km}$ for different equations of state. These results agree very well with ‘experimental results’ from observations of binary pulsars, which give neutron star masses of [Mølnvik and Østgaard, 1985, Prakash et al., 1988] $1.0\,M_\odot < M_{\max} < 2.2\,M_\odot$, or possibly [Glendenning, 1988, Taylor and Weisberg, 1989] $1.4\,M_\odot < M_{\max} < 1.8\,M_\odot$, and imply that stars calculated with stiff equations of state have greater maximum mass, lower central density and thicker crust than stars obtained with soft equations of state. At present, no reliable measurements of the radius of a neutron star exist. But general estimates give [Øvergård and Østgaard, 1991b, Prakash et al., 1988] $R \approx 10\,\mathrm{km}$.

In any case, theoretical calculations of the radius of neutron stars cannot be confirmed very well by observational data, and are more dependent than the total mass on the low-density equation of state.

Astronomical observations leading to global neutron star parameters such as total mass, radius, or moment of inertia, are important since they are sensitive to microscopic model calculations.

6 Effects that influence neutron stars

6.1 Superfluidity and superconductivity

Many-body fermion systems with an attractive effective interaction may favour formation of pairs of quasi-particles in two-body states and a phase transition to a superfluid state. If the particles are charged, the state will be superconducting. Since the basic nuclear interaction is attractive at large distances, a long range BCS pairing mechanism can arise in dense hadronic matter like neutron star matter. We then should get a superfluid which may undergo, for instance, viscous-free flow.

Superfluidity has little effect on properties of neutron stars like total mass, radius, etc., but we get some important physical consequences like thermal effects, magnetic effects, and hydrodynamic effects. The heat capacity of a superfluid is much lower than that of a normal degenerate gas at sufficiently low temperatures. This reduction in heat capacity should shorten the cooling time-scale for pulsars. However, the normal components of the fluid (matter) contribute fully to the heat capacity, and frictional interactions between normal and superfluid components can result in thermal dissipation of rotational energy and an increase in cooling time.

6.2 Thermal structure and cooling

Determination of surface temperatures of neutron stars by detecting thermal black-body radiation can, in principle, give significant information about the interior hadronic matter and neutron star structure.

Neutron stars are probably formed with very high interior temperatures $T > 10^{11}$ K in the core of a supernova explosion. The predominant cooling mechanism immediately after formation is neutrino emission with an initial cooling time-scale of seconds. In about a day, the internal temperature drops to about 10^9–10^{10} K. Neutrino cooling dominates for at least 1,000 years and probably much longer (100,000 years). Photon emission overtakes neutrino cooling only when the internal temperature falls to about 10^8 K, with a corresponding surface temperature of about 10^6 K.

Thermal evolution calculations are then sensitive to the chosen equation of state, the neutron star mass and radius, the magnetic field strength, the possible existence of superfluidity, pion condensation, quark matter, etc. Result simply potentially detectable photon emission first in the soft X-ray band, and young neutron stars may be detected as discrete X-ray sources (different from X-ray pulsars in binary systems accreting gas from their companions).

The first cooling period after the interior temperature has fallen below 10^{10} K is dominated by neutrino cooling, since any neutrinos emitted then may escape freely from the neutron star without interacting further with the neutron star matter. At very high temperatures $T > 10^9$ K the dominant neutrino process may be the URCA reactions [Østgaard, 2001]

6.3 Oscillations

The potentially most interesting of 'compact pulsators' are the neutron stars, but we know very little about possible excitation mechanisms, and lack any definitive observations of pulsating neutron stars. It is thus not clear yet whether or not neutron star pulsations have actually been observed, detections have been claimed for both pulsars and X-ray burst sources.

Neutron stars can sustain a variety of different oscillation modes. This diversity arises from different restoring forces that act on a displaced mass element; gravity, pressure gradients, elastic forces in the crustal material, magnetic fields, and centrifugal and Coriolis forces in rotating neutron stars (pulsars). The most important types of modes are the following:

(a) *p*-modes, with oscillation periods of $\Pi \sim 0.1$ ms. Pressure here acts as the main restoring force, and the eigenfunction is evenly distributed throughout the star.

(b) *f*-modes, with oscillation periods of $\Pi = 0.1$–0.8 ms. The number of radial nodes in the eigenfunction is zero.

(c) *g*-modes, with oscillation periods of $\Pi = 10$–400 ms. These modes are concentrated in the outer layers of the star, and the main restoring force is gravity.

(d) g^c-modes, which are another class of *g*-modes. The g^c-modes are concentrated in the fluid core, and similar types of modes are commonly known in geophysics as 'undertones.' They are very long-periodic, with oscillation periods of $\Pi > 10$ s. They exist in neutron stars with a solid crust, but vanish if the star is modeled as a homogeneous fluid.

(e) *r*-modes, which are retrograde waves with oscillation amplitudes along the surface. They are only present in rotating stars, and have oscillation periods of the same order as the rotation period. The restoring force is the Coriolis force.

(f) s^{br}-modes, which are essentially transverse shear waves largely confined to the crust. Their periods are shorter than $\sim$0.5–2 ms (depending on the model), and they are sensitive to the crust thickness.

(g) *i*-modes, which are interfacial modes that appear at the boundaries between the different layers of the neutron star. They are analogous to the terrestrial Rayleigh and Stoneley waves.

(h) t-modes, which are torsional oscillations confined solely to the crust. The fundamental mode of this series has periods of $\Pi \sim 20\,\mathrm{ms}$, while the higher overtones have progressively shorter periods starting at approximately 1 ms.

(i) s^{sf}-modes, which appear in superfluid matter, and have oscillation periods that are somewhat lower than the ordinary fluid modes.

6.4 Rotation

Pulsars rotate very rapidly with rotation periods almost down to 1 ms. Rotation is then obviously important to the star, and rotation effects are not negligible.

However, rotation complicates matter greatly. It affects both the oscillations and the equilibrium structure (through centrifugal deformation) of the star. The main part of the complicating effects results from the non-sphericity of the star. This is, however, a second order effect ($\sim \Omega^2$, where Ω is the angular frequency of rotation) which can be neglected for slow rotation.

Rotation and magnetic fields may, in any case, break the spherical symmetry and create (problematic) cylindrical symmetry ('mixing' with spherical symmetry). If we consider a small mass element, rotation gives rise to two new forces which will act on this mass element, i.e., centrifugal forces ($\sim \Omega^2$), and Coriolis forces ($\sim \Omega$). The Coriolis force is a first order effect, and cannot be neglected even for slow rotation. It does, however, affect only the oscillation, not the equilibrium structure of the star.

Entirely new phenomena can occur in a star that is both rotating and pulsating. Not only are the oscillation frequencies of the f-, p-, and g-modes modified, but new waves will be generated. Toroidal modes become important and give rise to Rossby-like waves with non-vanishing frequencies. These are called r-modes, and are retrograd waves with oscillation amplitudes along the surface. They are a purely rotational effect, which is connected to the conservation of vorticity.

6.5 Superstrong magnetic fields

In a magnetic field of the order of 10^{12}–10^{13} Gauss, which is characteristic of neutron star models for pulsars, the forms of matter are very different from those encountered on Earth. Because of the very strong tendency of electrons to move along magnetic field lines, an atom will have greatly increased binding energies [Skjervold and Østgaard, 1984, Skjervold and Østgaard, 1986] and cylindrical rather than spherical shapes. Molecules then take the form of linear chains of atoms parallel to the magnetic field lines. A strong chain-chain interaction arises from the large atomic quadrupole moments, and an approximately body-centered cubic crystal lattice is probably formed by parallel chains. This kind of crystals may exist at neutron star surfaces, where an iron lattice will be bound by about 10 keV per atom and a density of about $10^4\,\mathrm{g/cm^3}$. This should also have a great influence on the pulsar magnetospheric structure.

The magnetic field affects both the oscillations and the equilibrium structure of the star, and the effect is of second order ($\sim B^2$, where B is the magnetic field strength) for both cases. A detailed equilibrium model, therefore, is needed in order to study the influence of magnetic fields on stellar oscillations.

6.6 Glitches and starquakes

The most dramatic irregularities in pulsar periods are sudden spin-ups or glitches observed in the Crab and Vela pulsars, where a sudden decrease in period has been followed by an increase in the period derivative. Several models have been proposed to explain the origin of such spin-ups, and these include star-quakes in the crust or the core, superfluid vortex pinning, magnetospheric instabilities, and instabilities in the motions of the superfluid neutrons.

In the crust-quake model a sudden cracking of the neutron star crust decreases the moment of inertia and increases the angular rotation or frequency. If the nuclei in the crust of the neutron star form a solid Coulomb lattice, this crust is oblate in shape because of the rotation of the star. As the star slows down, centrifugal forces on the crust decrease, and stress arises because of the rigidity of the crust. The stress will finally reach a critical value and the crust cracks. Some stress is then relieved, the oblateness is reduced, the moment of inertia is suddenly decreased, and the angular frequency suddenly increased because of angular momentum conservation. Following a quake, the pulsar continues to slow down in the usual way until the stress builds up again to a critical value. The crust-quake model can be fitted to the Crab pulsar, but not to the large and frequent Vela spin-ups. The Vela glitches may then be explained by core-quakes or by pinned vorticity, i.e., vorticity jumps following sudden 'unpinning' of vortex lines in the pinned crustal neutron superfluid.

The Vela glitch may be explained by a simple 'two-component' neutron star model, which consists of a normal component (the crust and the charged particles) weakly coupled to the superfluid neutrons. The charged component is assumed to rotate at the observed pulsar frequency, since all charged particles are assumed to be strongly coupled to the magnetic field. The rotation of the neutron superfluid is assumed to be 'quasi-uniform' with a certain average angular frequency. The coupling between the two components is described by the relaxation time for frictional dissipation.

6.7 Gravitational radiation

General relativity predicts the possibility of gravitational waves, which should be 'ripples in the curvature of space-time' which propagate with the speed of light. Gravitational waves are not motions of a material medium, but they carry energy and momentum and can exert forces and do work. There is no dipole radiation in general relativity, so the lowest-order radiation is quadrupole.

We then consider gravitational radiation from binary systems, and the strongest evidence for the existence of gravitational waves may be the Hulse–Taylor binary pulsar PSR 1913+16 [Hulse and Taylor, 1975]. Apparent changes in the pulsar frequency may be explained by the Doppler effect due to orbital motion about an unseen companion star, and with all the system parameters known, we can predict a value for the change $\dot{P}$ in period because of gravitational radiation. If general relativity is all that is needed to explain the binary pulsar system, the quadrupole formula for gravitational wave emission may be confirmed to within the measurement (observation) errors. This can also be combined with two more general relativistic effects which include combinations of parameters to be measured: the periastron advance and combined second-order Doppler shift and gravitational redshift. Consistency of the relations between the key parameters of the system would confirm both the existence of gravitational waves and the absence of possible perturbing influences on the system which could complicate the interpretation of the data.

An axisymmetric object rotating rigidly about its symmetry axis has no time-varying quadrupole (or higher) moment and does not radiate gravitational waves. But it can radiate if the rotation axis is not the symmetry axis (or if it is non-axisymmetric). A possible example then would be a neutron star where the rigid crust could support a 'mountain,' or a symmetric neutron star rotating about a non-principal axis with a 'wobble.' A pulsar 'mountain' is, however, less than a few meters high.

The best 'source' of gravitational radiation should be, for instance, gravitational collapse of a massive star to a neutron star. The star then could radiate a substantial fraction of Mc^2 if the collapse is sufficiently non-spherical.

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